A Course Outline for Algebra 2

Developed by Ann Shannon & Associates
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ALGEBRA 2 COURSE OUTLINE

Content Area	Formative Assessment Lessons	# of Days
A0 Introduction: Mathematical Investigations		8
A1 Exponential Functions N-RN The Real Number System 1, 2 A-SSE Seeing Structure in Expressions 3c, 4 F-IF Interpreting Functions 3, 7e, 8b F-BF Building Functions 2, 4a F-LE Linear, Quadratic, and Exponential Models 2, 4, 5 N-RN 1,2; A-SSE 3c,4;F-BF 2 — F-IF 3,7e,8b; F-LE 2,4 — F-BF 4a; F-LE 5	Rational and Irrational Numbers 2	25
A2 Trigonometric Functions F-IF Interpreting Functions 7e F-TF Trigonometric Functions 1, 2, 5, 8 F-IF 7e — F-TF 1,2,5,8	Ferris Wheel	15
A3 Functions F-IF Interpreting Functions 4, 6. 7c, 9 F-BF Building Functions 1a, 1b, 3 F-LE Linear, Quadratic, and Exponential Models 2, 5 F-IF 4,6; F-BF 1a,1b — F-IF 7c,9; F-LE 2 — F-BF 3; F-LE 5		26
A4 Rational and Polynomial Expressions N-CN The Complex Number System 1, 2, 7 A-SSE Seeing Structure in Expressions 2 A-APR Arithmetic with Polynomials and Rational Expressions 2, 3, 4, 6 A-CED Creating Equations 1 A-REI Reasoning with Equations and Inequalities 1, 2, 4b, 6, 7, 11 G-GPE Expressing Geometric Properties with Equations 2 A-SSE 2;A-APR 2,3; A-REI 1,2,11 — A-APR 6;A-CED 1; A-REI 4b — N-CN 1,2,7;A-APR 4; A-REI 6,7;G-GPE 2	Representing Polynomials Manipulating Radicals	15
P1 Probability N-Q Quantities 2 S-CP Conditional Probability and the Rules of Probability 1, 2, 3, 4, 5, 6, 7 N-Q 2 — S-CP 1,2,3,4,5,6,7	Modeling Conditional Probabilities 1: Lucky Dip Modeling Conditional Probabilities 2 Medical Testing	28
S2 Statistics S-IC Making Inferences and Justifying Conclusions 1, 2, 3, 4, 5, 6 S-IC 3,4,5,6 — S-IC 1,2		27
Total days:		138

Most of the Formative Assessment Lessons are available at http://map.mathshell.org/materials/lessons.php

High School Overview

Algebra 1	Geometry	Algebra 2
A0 Introduction	G0 Introduction and Construction	A0 Introduction
A1 Modeling With Functions	G1 Basic Definitions and Rigid Motion	A1 Exponential Functions
A2 Linear Functions	G2 Geometric Relationships and Properties	A2 Trigonometric Functions
A3 Linear Equations & Inequalities in One Variable	G3 Similarity	A3 Functions
Modeling Unit 1	M1 Geometric Modeling 1	Modeling Unit 1
A4 Linear Equations & Inequalities in Two Variables	G4 Coordinate Geometry	A4 Rational and Polynomial Expressions
A5 Quadratic Functions	G5 Circles and Conics	P1 Probability
A6 Quadratic Equations	G6 Geometric Measurements and Dimensions	S2 Statistics
S1 Statistics	G7 Trigonometric Ratios	Modeling Unit 2
Modeling Unit 2	M2 Geometric Modeling 2	
1. Make sense of problems and persevere in solving them.	4. Model with mathematics.	7. Look for and make use of structure.
2. Reason abstractly and quantitatively.	5. Use appropriate tools strategically.	8. Look for and express regularity in repeated reasoning.
3. Construct viable arguments and critique the reasoning of others.	6. Attend to precision.	

A0 Introduction
8 Days

This introductory unit is designed to provide students with an opportunity to engage in worthwhile mathematical investigations; it has a number of related purposes. The most important of these is to let students draw heavily on the eight Mathematical Practices and establish their importance to this entire course. Through working with the mathematical practices, students will come to understanding the importance of seeing, doing, re-constructing, and supposing, in learning mathematics, and hopefully also realize that mathematics is not just facts to be memorized.

Examples of the kinds of mathematical investigations that students will work on during this unit can be found at Henri Picciotto's Mathematics Education Page (http://www.MathEducationPage.org/).

One of these investigations is called "Area on Graph Paper"; it explores the least and greatest perimeters that can be made with polyominoes of a fixed area. This calls on students to make and test hypotheses, be systematic, and draw conclusions. It will encourage students to deploy MP.1 (*Make sense of problems and persevere in solving them*) and MP.7 (*Look for and make use of structure*). (http://www.MathEducationPage.org/new-algebra/new-algebra.html)

The McNuggets Problem, another type of mathematical investigation, which can also be found on Henri's website in the document "Early Mathematics: A Proposal for New Directions" (http://www.mathedpage.org/early-math/early.html) is particularly valuable in an introductory unit such as this. Students are invited to consider the number of chicken nuggets that might be bought if a fast food store sells them in quantities of 6, 9, and 20. Students are asked to determine how many nuggets can and cannot be bought. Students are also invited to determine what is the largest number of nuggets that cannot be bought, and then prove that every number greater than this largest number can be bought. (http://www.MathEducationPage.org/early-math/early.html)

Another interesting investigation, titled "Which is Greater — Using Linear Functions," provides students with a number of linear equations written in slope intercept form. Students are asked to determine which of these would produce graphs that intersect to the left of the *y*-axis, to the right of the *y*-axis, or on the *y*-axis, or not intersect at all. Then students are asked to say how they might determine each of these cases without graphing. This genre of investigation will call upon students to put to work their learning from Grade 8. (http://www.MathEducationPage.org/calculator/which-is/greater-linear.pdf)

Another purpose of this introductory unit is to establish the norms for doing mathematics, and set the scene for the rest of the course. Teachers will be encouraged to enact these investigations in a way that enables students to take responsibility for their own learning and act as instructional resources for each other. The teacher will provide feedback that moves the learning forward and engineer effective discussion so as to facilitate learning. The intent of each mathematical investigation will be made explicit so as to provide a purpose for students.

The genius of this type of investigation is that, in a heterogeneous class of students, it is highly likely that every student will be able to do something, but also highly likely that no student will be able to do everything.

A1 Exponential Functions
25 Days

In this unit we draw upon standards from five clusters, namely: the real number system; seeing structure in expressions; building functions; interpreting functions; and modeling with linear, quadratic and exponential functions.

Students have been taught the real numbers system from grade 8 on. When constructing the concept of exponential functions, students' understanding of real numbers is needed in two places, the base number and the exponent. The definition of positive integer exponents as repeated multiplication of the base by itself is not useful when dealing with rational exponents. (If $3^2 = 3 \cdot 3$, then $3^{½}$ =?) This, and the fact that very often students are confused by fractions, should advise the kinds of activities we provide students in this unit. Being aware of the possible struggles, we should select activities that give students an opportunity to clearly develop conceptual understandings.

Point in question: If students develop the sequence $3^3 = 3 \cdot 3 \cdot 3 = 27$, $3^2 = 3 \cdot 3 = 9$, $3^1 = 3 = 3$, $3^0 = 1$, and understand that the pattern continues by dividing by 3 to obtain the value of the next term 3^{-1}, they can corroborate the definition of zero exponent for any non-zero number: $a^0 = 1$.

With further development of this sequence, $3^{-1} = 1/3$, $3^{-2} = 1/9$, $3^{-3} = 1/27$, students will be able to grasp with concrete examples the definition of a negative exponents as $x^{-n} = 1/x^n$ thus making this definition more conceptual for students after having analyzed sequences with different bases. Recognizing the geometric sequence here will help also with conceptual understanding for A-SSE 4 and F-IF 3.

Now to extend this understanding to the concept of $3^{½} = \sqrt{3}$, study the following example: $3^{½} \cdot 3^{½} = 3^{½ + ½} = 3^1$. By the law of exponents $x^a \cdot x^b = x^{a+b}$. We must also rely here on the student's conceptual understanding of the fact that $5 \cdot 5 = 25$ if and only if $\sqrt{25} = 5$, extending it to radicands that are not perfect squares. Students can then generalize their understanding of fractional exponents: $25^{½} = \sqrt{25}$, so $3^{½} = \sqrt{3}$, and furthermore $a^{½} = \sqrt{a}$ for $a \geq 0$.

It is important to ensure that students have practice with the distinction between $(-2)^4$ and -2^4 before asking them to rewrite expressions involving radicals and rational exponents, so as to avoid confusion and misunderstandings. When dealing with the properties of exponents, it is very important to pay attention to the parenthesis so as to see that $(a/b)^{-4} = a^{-4} \cdot b^{-4}$.

The Real Number System N-RN

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/2}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Students are probably familiar with the properties of operations that are not closed in a given set of numbers. Here we are extending students' understanding to explain that the sum and product of a rational and an irrational number are irrational. Students will practice their skills of algebraic manipulation to simplify expressions and derive formulas.

Seeing Structure in Expressions A-SSE

3c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{\frac{1}{12}})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

Interpreting Functions F-IF

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

It is important for students to gain a conceptual understanding of geometric and arithmetic sequences, and practice identifying them. This practice must include opportunities for students to identify the common ratio or common difference for each type of sequence. This is a basic foundation for extending the concepts to modeling with linear, quadratic, and exponential functions.

Linear, Quadratic, and Exponential Models F-LE

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
5. Interpret the parameters in a linear or exponential function in terms of a context.

Building Functions F-BF

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

4a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *for* $x > 0$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.

Formative Assessment Lessons

Rational and Irrational Numbers 2

This lesson is also about rational and irrational numbers. Students are asked for examples of rational and irrational numbers, and for a definition in their own terms. Students need to explore what happens when adding, subtracting, multiplying or dividing two rational numbers, two irrational numbers, or one of each. They need to prove or disprove statements about the resulting numbers by using the fact that the rational numbers are closed under the arithmetic operations, using contradiction arguments, and finding counterexamples. They also need to consider lengths and areas of geometric objects like circles, right triangles, and rectangles, and find examples of when one or more of these measures can be rational or irrational.

Students are also given cards with various statements related to rational and irrational numbers, and asked to sort the statements into three categories: always true, sometimes true and never true.

Teaching and Assessment Resources

Simplifying expressions involving radicals and rational exponents using the properties of exponents: http://www.kutasoftware.com

Identifying geometric sequences and deriving the formula for the sum of a finite geometric series: http://www.mathguide.com/lessons/SequenceGeometric.html

Practice with sequences as functions, sums of geometric sequences and more by Anita Wah and Henri Picciotto: http://www.MathEducationPage.org/attc/seq/geometric.pdf (Geometric Sequences and Series can be downloaded from www.MathEducationPage.org for non-commercial use, as long as the authors are credited and the Web site referenced. See www.MathEducationPage.org/rights for information on derivative works.)

http://www.MathEducationPage.org/

http://illustrativemathematics.org/

The following tasks can be found at http://map.mathshell.org/materials/tasks.php :

Novice Tasks	Apprentice Tasks	Expert Tasks
N01: The Real Number System		

A2 Trigonometric Functions
15 Days

In this unit it is important to give students the conceptual understanding of a periodic functions as a function that repeats its values in regular intervals also called period. This is a solid foundation for the development of all trigonometric functions and to provide students with a clear understanding of the meaning of amplitude and frequency as indicators of the behavior of the curve. The depth of this unit brings students beyond the trigonometric ratios and their application to solving triangles into understanding the use of the unit circle to study the entire function and the effect of changing parameters.

Understanding angle measures in radians and its usefulness is key here, since radians will be especially useful in Calculus. But it also noteworthy that computer programs use radians by default and care must be taken when using the graphing calculator to select radians or angles to avoid mistakes.

The unit circle is a tool to express the relationship between angle measure and the coordinate pair (x, y) as cos and sin of the angle.

Trigonometric Functions F-TF

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.

An extension of the Pythagorean Theorem studied before allows students to prove the identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

Interpreting Functions F-IF

7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Formative Assessment Lessons

Ferris Wheel

In this lesson, students model how the height above the ground of a Ferris wheel rider changes over time as the wheel turns at a constant rate. Students need to use a trigonometric function to model this behavior, in this case, a cosine function. They are given the relevant information about the Ferris wheel: the height of the axel, the diameter of the wheel, the starting point of the rider, and the period of the rotation. Students are then asked to convert these values into the basic parameters of a sinusoid: amplitude, period, phase and offset. They use this information to graph the position of the rider over time.

The goal of this lesson is to improve students' understanding of the relationships between the physical properties of rotating objects and the mathematical parameters of trigonometric functions.

Students are then paired and asked to match expressions of several cosine functions of varying amplitudes, offsets, and periods with corresponding graphs (if any graph is missing, they need to draw it). Finally, they are asked to match these with corresponding descriptions of several Ferris wheels of varying diameters, axle heights, and periods of revolution. In order to accomplish these tasks, students should already be familiar with the sine and cosine functions and with how the parameters of amplitude, frequency, and shifts affect their graphs. They also need to know that one can model the height of a point on a circle turning at a constant rate as a cosine (or sine) function. Ideally, they should know that the *x*- and *y*- coordinates of a point on the unit circle are the cosine and sine (respectively) of the angle between a radial line through that point and the positive *x*-axis.

Teaching and Assessment Resources

NCTM Illuminations activities: Graphs from the Unit Circle and Trigonometric Graphing

http://illuminations.nctm.org

http://www.MathEducationPage.org/

http://illustrativemathematics.org/

The following tasks can be found at http://map.mathshell.org/materials/tasks.php :

Novice Tasks	Apprentice Tasks	Expert Tasks
N09: Trigonometric Functions		

A3 Functions
26 Days

In this unit on functions we need to build on students' prior understanding of a function to become more comfortable with the similarities and differences between linear, quadratic, and exponential functions and their graphs. In the basic sense, a function represents the mathematical process of showing a relationship between an input and an output. Real-life examples can be used to build and interpret functions and as a tool to expand this basic definition to meaningful applications of modeling with functions.

Building Functions F-BF

1a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

1b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Linear, Quadratic, and Exponential Models F-LE

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
5. Interpret the parameters in a linear or exponential function in terms of a context.

Here we reinforce again the multiple representations of functions: verbal description, table, and graph; we emphasize the key features that convey information about the behavior of the function. Students need to acquire fluency in graphing several types of functions, namely: square root, cube root, absolute value, and identifying key individual features like symmetries, maximums and minimums, and intercepts, which allow us to sketch a curve, understanding its behavior.

The application of factorization to polynomial functions allows students to find meaning as they identify the zeroes of the function using factored forms whenever available.

Interpreting Functions F-IF

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

7c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal description). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Teaching and Assessment Resources

NCTM Illuminations Building Connections:
http://illuminations.nctm.org/LessonDetail.aspx?id=L282

http://map.mathshell.org/materials/index.php

http://www.MathEducationPage.org/

http://illustrativemathematics.org/

A4 Rational and Polynomial Expressions
15 Days

This unit will build on students' prior knowledge of algebraic expressions and identifying algebraic terms, as well as rewriting expressions in factored forms. Students have been exposed to the zero product property (A·B = 0 if and only if A = 0 or B = 0) when solving factorable quadratic equations. These will support students as we expand to the concept that a polynomial we can't factor can nevertheless be expressed as a product of factors added to a remainder. This allows us to arrive at the remainder theorem, to identify the zeroes of the polynomial, and to sketch its graph.

The Complex Number System N-CN

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
7. Solve quadratic equations with real coefficients that have complex solutions.

Seeing Structure in Expressions A-SSE

2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 – y^4$ *as* $(x^2)^2 – (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 – y^2)(x^2 + y^2)$.

As an integration of the experiences with algebraic manipulations and the properties or operations, special attention should be given here to extending these to the four operations with polynomials and to seeing that all operations are closed.

Arithmetic with Polynomials and Rational Expressions A-APR

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Creating Equations A-CED

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Expressing Geometric Properties with Equations G-GPE

2. Derive the equation of a parabola given a focus and directrix.

Reasoning with Equations and Inequalities A-REI

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

4b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$*

11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Formative Assessment Lessons

Representing Polynomials

Apart from a short consideration of linear and quadratic functions in the teacher guide and projector materials, this lesson deals exclusively with cubic functions. They are used to illustrate the relationships between zeros, factors, equations, and graphs, and between shifts (vertical and horizontal) and reflections (across the x-axis and the y-axis) and the corresponding changes in the algebraic expressions of the (cubic) functions. Students need to understand and apply these ideas in order to answer several questions, match equations to graphs (sometimes more than one equation can correspond to the same graph), graph equations and change expressions in order to produce shifts or reflections. Students will also decide if statements dealing with zeros, factors, values at other values of x, and transformations (shifts and reflections) are true or false.

Students need to know that a constant a is a zero of a polynomial if and only if $(x - a)$ is a factor of the equation. They need to understand how double or triple roots affect the graph. They need to understand that changing x to $(x - k)$ in $y = f(x)$ results in a horizontal shift, changing $f(x)$ to $f(x) + k$ results in a vertical shift, changing $f(x)$ to $-f(x)$ reflects the graph over the x-axis, and changing x to $-x$ reflects the graph over the y-axis.

Manipulating Radicals

In this lesson, students are given several equations in one variable involving square roots (on two occasions, a second variable is used). In each case, they need to solve the equation and find whether the solution set is empty (no solutions), a proper subset of the real numbers (in the examples given there is only one or sometimes only two solutions), or all real numbers (the equation is an identity). These cases are referred to as saying that the statements given by the equations are never true, sometimes true, or always true.

In order to accomplish their task, students need to know about the laws of exponents, especially squares and square roots (exponent ½). They also need to know how to square binomials, how squaring and taking square roots are related, and how to isolate square roots and square expressions in an equation in order to solve it. There is also an optional extension, consisting of the last two equations provided, that explores the imaginary number *i*, the square root of −1. But issues of the domain of a function are not explored in any significant depth.

Teaching and Assessment Resources

Investigation in four lessons involving use of polynomials, rational expressions, and different factored forms, tied together with statistical interpretation of data:

NCTM Illuminations: Whelk-Come to Mathematics :
http://illuminations.nctm.org/LessonDetail.aspx?ID=L483

http://www.MathEducationPage.org/ http://illustrativemathematics.org/

The following tasks can be found at http://map.mathshell.org/materials/tasks.php :

Novice Tasks	Apprentice Tasks	Expert Tasks
N02: Seeing Structure in Expressions	A17: Cubic Graph	
N03: Arithmetic with Polynomials and Rational Expressions		

P1 Probability
28 Days

This unit builds a foundation for working with probabilities. Basic rules of probability are discussed, and also conditional probabilities. Extra care should be taken to develop the conceptual understanding of probability, rather than just memorizing a relationship. Instruction through the use of simulations will help build the necessary conceptual foundation for the successful study of probability. The idea of independence is important and often misunderstood. Being able to explain probability in everyday language as well as interpret everyday language into probability statements is a critical skill for mastering these standards. Students should be exposed to multiple representations of information and asked to calculate probabilities from them. Two-way tables, Venn diagrams, and probability trees should be common in this curriculum.

Quantities N-Q

2. Define appropriate quantities for the purposes of descriptive modeling.

Conditional Probability and the Rules of Probability S-CP

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
6. Find the conditional probability of *A* given *B* as the fraction of *B*’s outcomes that also belong to *A,* and interpret the answer in terms of the model.
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

Formative Assessment Lessons

Modeling Conditional Probabilities 1: *Lucky Dip*

This is a lesson about conditional probability focusing on a single problem, a game in which two balls are randomly selected without replacement from a bag containing 3 black and 3 white balls. Students need to find the probability of selecting two balls of the same color. Students need to solve the problem, discuss each other’s solutions, and analyze and critique three given sample solutions with different misconceptions. In order to access the problem, they need to know and build on the basic computation of probabilities given finite sample spaces where the outcomes are equally likely. They need to be able to represent the sample space and all its outcomes, or make probability trees, and they need to interpret such models in the sample solutions provided and find where they are flawed. Also, different models emerge depending on whether the student thinks of taking two balls at once, or of taking one ball after the other.

Modeling Conditional Probabilities 2

In this lesson, students are asked to consider a series of extensions of the problem of *Modeling Conditional Probabilities 1: Lucky Dip*. First, given a total of 9 balls, they need to find the two (symmetrical) solutions where the game would be fair, i.e., the number of white and black balls so that the probability of choosing two of the same color equals the probability of choosing two of opposite color. Then they are given different initial numbers of black balls, and they are asked to find the number of white balls they need to add in order to make the game fair. As it turns out, sometimes there is a unique solution, sometimes there are two, and sometimes there are no solutions. Finally, they are asked to find general relationships among the number of black balls, white balls, and their sum if the game is fair.

Students need to represent the sample space, count the outcomes with same color and different color, and equate these numbers in order to make the game fair. In order to solve the more general or demanding extensions, they will need to introduce literal symbols and use algebra, eventually leading them to solve quadratic equations (which can be factored easily when there are solutions). For the simpler tasks, they may also be able to obtain solutions by guess-and-check methods. Whether or not they can solve the problem in its most general, abstract form, they should notice some patterns and relationships provided by the numerical solutions to various examples. These patterns involve perfect squares and triangular numbers, which should be familiar to them.

Medical Testing

This is a lesson about conditional probability in the context of false positives in medical testing. Students are given the probability that a person without a certain disease will test positive (and they are told that a person with the disease always tests positive). Then they are asked to compare two samples of 1000 people, each with a different percent of people having the disease. For each sample, they need to compute the number of people having and not having the disease, and the number of people testing positive and negative. Finally, for each sample, they need to compute the probability of getting a false positive. For this, they need to realize that they need to compute the ratio of the number of people not having the disease and testing positive to the number of people testing positive (this last number, of course, includes all the people who actually have the disease).

Students need to make sense of the “false positive paradox,” namely that the probability of getting an incorrect positive test depends not only on the accuracy of the test but also on the rarity of the disease in the population. The rarer the disease is, the higher the probability that a positive test is wrong.

Students can solve the problem by modeling it in a variety of ways, and they are also asked to analyze given sample student work with varying approaches, including Venn diagrams, probability trees, and algebraic solutions.

Teaching and Assessment Resources

NCTM Virtual Spinner: an applet for simulating experimental probabilities to compare to theoretical probabilities. http://illuminations.nctm.org/ActivityDetail.aspx?ID=79

Let's Make a Deal – an applet for simulating the famous "Monty Hall" problem involving conditional probability.
http://nlvm.usu.edu/en/nav/frames_asid_117_g_4_t_5.html?from=category_g_4_t_5.html

http://map.mathshell.org/materials/index.php

http://www.MathEducationPage.org/

http://illustrativemathematics.org/

S2 Statistics
27 Days

In this unit, students will explore the process of making inferences about populations. This unit relies heavily on vocabulary and conceptual understandings. Simulation is also a critical part of developing inferences. Students should spend time learning about the various data collection instruments by constructing and carrying them out for themselves. Comparisons between collected data and simulations will lend a real-life feel to this unit. Overall, students should see and understand that statistics is a process for making inferences from a population by examining a random sample; and they should realize why this is useful.

Making Inferences and Justifying Conclusions S-IC

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*
3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

Teaching and Assessment Resources

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