# A Summary of Course Outlines for Middle School

Developed by Ann Shannon & Associates
for the Bill & Melinda Gates Foundation

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# OVERVIEW OF MIDDLE SCHOOL COURSE OUTLINES



Green indicates a *major* cluster, yellow indicates an *additional* cluster, and blue indicates a *supporting* cluster, as given in the Cluster Emphases from the PARCC and Smarter Balanced Consortia.

| Grade 6 | Grade 7 | Grade 8 |
|---|---|---|
| 6.RP Understand ratio concepts and use ratio reasoning to solve problems. | 7.RP Analyze proportional relationships and use them to solve real-world and mathematical problems. | 8.NS Know that there are numbers that are not rational, and approximate them by rational numbers. |
| 6.NS Apply and extend previous understandings of multiplication and division to divide fractions by fractions. | 7.NS Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. | 8.EE Work with radicals and integer exponents. |
| 6.NS Compute fluently with multi-digit numbers and find common factors and multiples. | 7.EE Use properties of operations to generate equivalent expressions. | 8.EE Understand the connections between proportional relationships, lines, and linear equations. |
| 6. NS Apply and extend previous understandings of numbers to the system of rational numbers. | 7.EE Solve real-life and mathematical problems using numerical and algebraic equations | 8.EE Analyze and solve linear equations and pairs of simultaneous linear equations. |
| 6.EE Apply and extend previous understandings of arithmetic to algebraic expressions. | 7.G Draw, construct, and describe geometrical figures and describe the relationships between them. | 8.F Define, evaluate, and compare functions. |
| 6.EE Reason about and solve one-variable equations and inequalities. | 7.G Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. | 8.F Use functions to model relationships between quantities. |
| 6.EE Represent and analyze quantitative relationships between dependent and independent variables. | 7.SP Use random sampling to draw inferences about a population. | 8.G Understand congruence and similarity using physical models, transparencies, or geometry software. |
| 6.G Solve real-world and mathematical problems involving area, surface area, and volume. | 7.SP Draw informal comparative inferences about two populations. | 8.G Understand and apply the Pythagorean theorem. |
| 6.SP Develop understanding of statistical variability. | 7.SP Investigate chance processes and develop, use, and evaluate probability models. | 8.G Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. |
| 6.SP Summarize and describe distributions. | | 8.SP Investigate patterns of association in bivariate data. |

**Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

# Grade 6-8 Unit Overview

The colored bars reflect the weighting of the standards within each unit based on the Cluster Emphases given by the PARCC and Smarter Balanced Consortia.

| Grade 6 | Grade 7 | Grade 8 |
|---|---|---|
| Unit 6.0 Introduction: Mathematical Investigations | Unit 7.0 Introduction: Mathematical Investigations | Unit 8.0 Introduction: Mathematical Investigations |
| Unit 6.1 Ratio and Proportional Relationships<br>6.RP.1, 2, 3 | Unit 7.1 Ratio and Proportional Relationships<br>7.RP.1 ,2 ,3 | Unit 8.1 Numerical Expressions<br>8.EE.1, 2, 3, 4 |
| Unit 6.2 Critical Area 1<br>6.RP | Unit 7.2 Critical Area 1<br>7.RP | Unit 8.2 Proportional Relationships, Lines, and Equations<br>8.EE.5, 6 |
| Unit 6.3 Rational Number System<br>6.NS.5, 6, 7, 8<br>6.G.3 | Unit 7.3 The Number System<br>7.NS.1 ,2 ,3 | Unit 8.3 Linear Equations and Systems of Linear Equations<br>8.EE.7, 8 |
| Unit 6.4 Division of Fractions by Fractions<br>6.NS.1<br>6.NS.2, 3 | Unit 7.4 Complex problem solving<br>7.RP.3; 7.NS.3 | Unit 8.4 Bivariate Data<br>8.SP.1, 2, 3, 4 |
| Unit 6.5 Critical Areas 1 & 2<br>6.RP; 6.NS.1<br>6.NS.2, 3 | Unit 7.5 Expressions<br>7.EE.1 ,2 ,3 | Unit 8.5 Critical Area 1<br>8.EE<br>8.SP |
| Unit 6.6 Expressions<br>6.EE.1, 2, 3, 4<br>6.NS.4 | Unit 7.6 Equations<br>7.EE.3, 4 | Unit 8.6 Functions<br>8.F.1, 2, 3<br>8.F.4, 5 |
| Unit 6.7 Equations and Inequalities<br>6.EE.5, 6, 7, 8 | Unit 7.7 Critical Areas 1 & 2<br>7.RP; 7.NS; 7.EE | Unit 8.7 Critical Areas 1 & 2<br>8.EE; 8.F.1, 2, 3<br>8.F.4, 5; 8.SP |
| Unit 6.8 Quantitative Relationships<br>6.EE.9; 6.NS.8<br>6.G.2<br>6.SP.4 | Unit 7.8 Geometric Figures<br>7.G.1, 2, 3 | Unit 8.8 Rational and Irrational Numbers<br>8.NS.1, 2 |
| Unit 6.9 Critical Areas 1, 2, & 3<br>6.RP; 6.EE; 6.NS.1<br>6.NS.2, 3 | Unit 7.9 Geometric Measurement<br>7.G.4, 5, 6 | Unit 8.9 Transformations<br>8.G.1, 2, 3, 4, 5 |
| Unit 6.10 Statistics<br>6.SP.1, 2, 3, 4, 5 | Unit 7.10 Critical Areas 1, 2, & 3<br>7.RP; 7.EE; 7.NS<br>7.G | Unit 8.10 Pythagorean Theorem<br>8.G.6, 7, 8 |
| Unit 6.11 Geometry<br>6.G.1, 2, 3, 4 | Unit 7.11 Statistics<br>7.SP.1, 2<br>7.SP.3, 4 | Unit 8.11 Volume of Cones, Cylinders, and Spheres<br>8.G.9 |
| Unit 6.12 Critical Areas 1, 2, 3, & 4<br>6.RP; 6.EE; 6.NS.1<br>6.NS.2, 3; 6.SP | Unit 7.12 Probability<br>7.SP.5, 6, 7, 8 | Unit 8.12 Critical Areas 1, 2, & 3<br>8.EE; 8.F.1, 2, 3; 8.G.1-8<br>8.F.4, 5; 8.SP<br>8.G.9 |
| | Unit 7.13 Critical Areas 1, 2, 3, & 4<br>7.RP; 7.NS; 7.EE<br>7.SP.1, 2, 5-8<br>7.G; 7.SP.3, 4 | |

## CCSS Tools: Progressions

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Source: http://ime.math.arizona.edu/progressions/

| Grade 6 | Grade 7 | Grade 8 |
|---|---|---|
| 6.RP Understand ratio concepts and use ratio reasoning to solve problems. | 7.RP Analyze proportional relationships and use them to solve real-world and mathematical problems. | 8.EE Understand the connections between proportional relationships, lines, and linear equations. |
| 6.EE Represent and analyze quantitative relationships between dependent and independent variables. | | 8.F Define, evaluate, and compare functions.<br>8.F Use functions to model relationships between quantities. |
| 6.EE Apply and extend previous understandings of arithmetic to algebraic expressions.<br>6.EE Reason about and solve one-variable equations and inequalities. | 7.EE Use properties of operations to generate equivalent expressions.<br>7.EE Solve real-life and mathematical problems using numerical and algebraic equations. | 8.EE Work with radicals and integer exponents.<br>8.EE Analyze and solve linear equations and pairs of simultaneous linear equations. |
| 6.NS Apply and extend previous understandings of numbers to the system of rational numbers.<br>6.NS Apply and extend previous understandings of multiplication and division to divide fractions by fractions.<br>6.NS Compute fluently with multi-digit numbers and find common factors and multiples. | 7.NS Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. | 8.NS Know that there are numbers that are not rational, and approximate them by rational numbers. |
| 6.G Solve real-world and mathematical problems involving area, surface area, and volume. | 7.G Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.<br>7.G Draw, construct, and describe geometrical figures and describe the relationships between them. | 8.G Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.<br>8.G Understand congruence and similarity using physical models, transparencies, or geometry software.<br>8.G Understand and apply the Pythagorean theorem. |
| 6.SP Develop understanding of statistical variability.<br>6.SP Summarize and describe distributions. | 7.SP Use random sampling to draw inferences about a population.<br>7.SP Draw informal comparative inferences about two populations.<br>7.SP Investigate chance processes and develop, use, and evaluate probability models. | 8.SP Investigate patterns of association in bivariate data. |

## Formative Assessment Lesson Overview

| Grade 6 | Grade 7 | Grade 8 |
|---|---|---|
| Sharing Costs: Traveling to School<br>Optimizing: Security Cameras<br>Laws of Arithmetic<br>Mean, Median, Mode, and Range<br>Design a Candy Carton | Developing a Sense of Scale<br>Proportion and Non-proportion Situations<br>Using Positive and Negative Numbers in Context<br>Drawing to Scale: Designing a Garden<br>Steps to Solving Equations<br>Increasing and Decreasing Quantities by a Percent<br>Using Dimensions: Designing a Sports Bag<br>Maximizing Area: Gold Rush<br>Estimations and Approximations: The Money Munchers<br>Estimating: Counting Trees<br>Evaluating Statements about Probability | Estimating Length Using Scientific Notation<br>Generalizing Patterns: The Difference of Two Squares<br>Lines and Linear Equations<br>Solving Linear Equations in One Variable<br>Classifying Solutions to Systems of Equations<br>Solving Real-Life Problems: Baseball Jerseys<br>Interpreting Distance-Time Graphs<br>Modeling Situations with Linear Equations<br>Repeating Decimals<br>Identifying Similar Triangles<br>The Pythagorean Theorem: Square Areas<br>Modeling: Making Matchsticks |

The Formative Assessment Lessons are available at [http://map.mathshell.org/materials/lessons.php](http://map.mathshell.org/materials/lessons.php)

# Grade 6 Course Outline

The colored bars reflect the weighting of the standards within each unit based on the Cluster Emphases given by the PARCC and Smarter Balanced Consortia.

| Content Area | Formative Assessment Lessons | # of Days |
|---|---|---|
| Unit 6.0 Introduction: Mathematical Investigations | | 5 |
| Unit 6.1 Ratio and Proportional Relationships<br>6.RP.1, 2, 3 | Sharing Costs: Traveling to School<br>Optimizing: Security Cameras | 10 |
| Unit 6.2 Critical Area 1<br>6.RP | | 5 |
| Unit 6.3 Rational Number System<br>6.NS.5, 6, 7, 8 / 6.G.3 | | 10 |
| Unit 6.4 Division of Fractions by Fractions<br>6.NS.1 / 6.NS.2, 3 | | 10 |
| Unit 6.5 Critical Areas 1 & 2<br>6.RP; 6.NS.1 / 6.NS.2, 3 | | 13 |
| Unit 6.6 Expressions<br>6.EE.1, 2, 3, 4 / 6.NS.4 | Laws of Arithmetic | 10 |
| Unit 6.7 Equations and Inequalities<br>6.EE.5, 6, 7, 8 | | 10 |
| Unit 6.8 Quantitative Relationships<br>6.EE.9; 6.NS.8 / 6.G.2 / 6.SP.4 | | 10 |
| Unit 6.9 Critical Areas 1, 2, & 3<br>6.RP; 6.EE; 6.NS.1 / 6.NS.2, 3 | | 18 |
| Unit 6.10 Statistics<br>6.SP.1, 2, 3, 4, 5 | Mean, Median, Mode, and Range | 10 |
| Unit 6.11 Geometry<br>6.G.1, 2, 3, 4 | Design a Candy Carton | 9 |
| Unit 6.12 Critical Areas 1, 2, 3, & 4<br>6.RP; 6.EE; 6.NS.1 / 6.NS.2, 3; 6.SP | | 24 |
| Total Days | | 144 |

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# Formative Assessment Lessons Aligned to Grade 6

## Unit 6.1 Ratio and Proportional Relationships

**Sharing Costs: Travelling to School**

In this lesson, students use proportional reasoning to determine how to share the cost of gasoline for a car ride to and from school (for the whole school term). Students get picked up and dropped off at their houses along the way, so each student travels a different distance, and there are different numbers of students in the car during different parts of the ride.

Students need to interpret a given map to scale to figure out the proportional relationships among the distances traveled, as well as how many students are in the car for each part of the ride. They may or may not realize that they can simply double their numbers for one leg of the trip without changing the proportions.

Students need to defend their method for sharing the cost. There is more than one possible result based on how to interpret what is fair. One method may simply divide up the cost in proportion to the distance traveled by each student, while another method may consider sharing the cost between the students in the car during each part of the trip, after fixing a total cost per unit distance.

In order to succeed in this lesson, students need to model the situation, interpret a map, organize their data, and manipulate part-to part and part-to-whole ratios.

**Optimizing: Security Cameras**

In this lesson, students draw lines of sight in order to determine the parts of a store (in the shape of a “fat square letter C”, which is a non-convex polygon with right angles) can and cannot be seen by a security camera. The store is subdivided into 20 congruent squares. The locations of ten people within the store are given, and the student needs to determine which people the camera can see. They need to find the percent of the area of the store that cannot be seen. Students compute areas and percentages by counting squares and computing the areas of partial regions of the squares.

Next, students need to change the location of the camera and choose a new location that will minimize the hidden area. They need to justify their solution, and it turns out that for a complete justification, they should discover and argue that there is a segment along which any position results in the same optimal camera coverage. This can be shown by demonstrating that the regions that are hidden can be joined to form a triangle and this triangle has the same area regardless of the position of the camera along this optimal segment.

For this lesson, students need to interpret a model of a real-life problem, draw auxiliary lines (lines of sight), be able to compute the area of regions made up of squares and triangles, be able to visualize joining two regions to form a triangle or square, and be able to compute the area of a triangle from its base and height. They also need to use proportional reasoning and basic arithmetic to find the percentage of areas that cannot be seen by the camera.

## Unit 6.6 Expressions

### Laws of Arithmetic

This lesson is about making use of numerical expressions to represent the area of compound regions formed by one or more rectangles, and drawing one or more such regions whose area can be represented by a given numerical expression. The goal is for students to develop an understanding of the relationship between geometric figures and numerical expressions that describe their areas. The lesson also seeks to improve students' understanding of the commutative and distributive properties, and to reinforce their grasp of order of operations, including the role of parenthesis and notational conventions with symbols like the fraction bar.

Students are asked to select from a number of options which numerical expressions represents the area of a given region and which region has an area that is expressed by a given numerical expression. They are also asked to produce one or more numerical expressions representing the area of a given region, and vice-versa. Finally, there is a matching task where they have to match expressions to regions. Some cards are missing expressions or regions, for these cards the students will fill in expressions or regions that match an existing card from another set.

The matching cards are designed so that students can also strengthen their understanding of the distinction between the sum of two squares and the square of the sum.

In order to successfully complete this lesson, students need to know and understand the formula for the area of a rectangle, be able to add areas of adjacent regions, break up a region into rectangular components, and have some fluency with the basic arithmetic operations.

## Unit 6.10 Statistics

### Mean, Median, Mode, and Range

The goal of this lesson is for students to understand the relationship between frequency charts, frequency tables and the statistical measures of shape and central tendency that describe them. Given a frequency chart, the students will be asked to find these descriptors, as well as gathering other information from the chart, such as how many data points are in the sample set. Conversely, students are given the four statistical measures and asked to create a frequency chart that can be described by those measures. Students should learn from this that while a frequency chart has only one set of statistical measures, a given set of statistical measures can describe many different frequency charts.

Students are also asked to match several frequency charts with cards listing the four statistical measures. The students will work in small groups to compare and improve on their answers. They will be encouraged to explain each other's work to ensure that the entire group is improving.

In order to engage in this lesson, students need to know and understand the definitions of mean, median, mode and range, as well as how to find or compute them. They also need a basic understanding of frequency charts: they must know how to find the number of data points and how to interpret the axes.

**Design a Candy Carton**

This lesson asks students to design a carton that will hold 18 cylindrical pieces of candy 1 cm in height and 2 cm in diameter. It is not an optimization problem (students are not asked to minimize the surface or volume of the carton, for example). Students need to create two different cartons and compare them to each other and three sample designs. They must explain why the candies fit inside and consider different arrangements of the 18 pieces of candy. They must keep in mind packing issues, like the fact that there will be space between the candies.

Students need to use rulers and measure with metric units to design, draw and cut out nets for solids. Depending on their design, they may need to use a protractor. They need to visualize whether a net will close into a prism or not, and how. They also need to design flaps in appropriate places in order to glue the different faces together.

No restrictions are given for the design, but the expectation and the sample solutions given for discussion all involve rectangular or triangular prisms. Students evaluate designs for their shape, reasonable dimensions, and how well they may stack in a store shelf.

# Grade 7 Course Outline

The colored bars reflect the weighting of the standards within each unit based on the Cluster Emphases given by the PARCC and Smarter Balanced Consortia.

| Content Area | Formative Assessment Lessons | # of Days |
|---|---|---|
| Unit 7.0 Introduction: Mathematical Investigations | | 5 |
| Unit 7.1 Ratio and Proportional Relationships<br>7.RP.1 ,2 ,3 | Developing a Sense of Scale<br>Proportion and Non-proportion Situations | 10 |
| Unit 7.2 Critical Area 1<br>7.RP | | 5 |
| Unit 7.3 The Number System<br>7.NS.1 ,2 ,3 | Using Positive and Negative Numbers in Context | 10 |
| Unit 7.4 Complex problem solving<br>7.RP.3; 7.NS.3 | | 10 |
| Unit 7.5 Expressions<br>7.EE.1 ,2 ,3 | Drawing to Scale: Designing a Garden | 10 |
| Unit 7.6 Equations<br>7.EE.3, 4 | Steps to Solving Equations | 10 |
| Unit 7.7 Critical Areas 1 & 2<br>7.RP; 7.NS; 7.EE | Increasing and Decreasing Quantities by a Percent | 14 |
| Unit 7.8 Geometric Figures<br>7.G.1, 2, 3 | | 10 |
| Unit 7.9 Geometric Measurement<br>7.G.4, 5, 6 | Using Dimensions: Designing a Sports Bag<br>Maximizing Area: Gold Rush | 10 |
| Unit 7.10 Critical Areas 1, 2, & 3<br>7.RP; 7.EE; 7.NS / 7.G | Estimations and Approximations: The Money Munchers | 17 |
| Unit 7.11 Statistics<br>7.SP.1, 2 / 7.SP.3, 4 | | 5 |
| Unit 7.12 Probability<br>7.SP.5, 6, 7, 8 | | 5 |
| Unit 7.13 Critical Areas 1, 2, 3, & 4<br>7.RP; 7.NS; 7.EE / 7.SP.1, 2, 5-8 / 7.G; 7.SP.3, 4 | Estimating: Counting Trees<br>Evaluating Statements about Probability | 23 |
| Total Days | | 144 |

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# Formative Assessment Lessons Aligned to Grade 7

## Unit 7.1 Ratio and Proportional Relationships

### Developing a Sense of Scale

This is a lesson about proportional reasoning. Students need to scale quantities in different contexts, such as amounts of ingredients in recipes, lengths of enlargements of pictures, and prices of different sizes of paint cans. Although students can set up proportions to solve the problems, this method is inefficient, and the objective is to understand the multiplicative nature of these situations and the meaning and usefulness of the unit rate or constant of proportionality in each context. Students must especially distinguish between multiplicative and additive relationships in one of the given sample solutions they are asked to analyze. They must also realize that multiplying by a number with absolute value less than 1 results in a smaller (absolute value) number.

Students need to work in small groups, compare and improve solutions, explain and critique each other's work, and analyze given sample solutions.

### Proportion and Non-proportion Situations

This lesson is about proportional relationships, their characteristics, and the ability to recognize proportional versus non-proportional situations. Students are given several real-life scenarios where two quantities are related. They need to reason about each situation and do some computation in order to answer specific questions. Next, for each situation, they need to decide whether it is true or not that when one of the quantities is doubled, so is the other, and whether it is true or not that the ratio of the two quantities is invariant. They need to explain their answers. Some explanations may involve interpreting the constant of proportionality or unit rate.

Students are also given a number of incomplete problems of a similar nature, which they complete by choosing missing values and then solving as before.

The relationships found in these situations are all linear or inverse proportions, with some examples where there is a break or a discrete context. Some of the linear relationships are proportional and some have a nonzero y-intercept.

Students need to describe and explain the properties of proportional relationships: the graph is a line through the origin; the ratio of the quantities is constant; one quantity is a multiple of the other; if one is zero, so is the other; doubling one results in doubling the other.

In order to engage in the lesson, students need to interpret and model real-life scenarios, use proportional reasoning, have some familiarity with linear functions and their graphs, and solve simple equations including proportions and more general linear equations.

## Unit 7.3 The Number System

**Using Positive and Negative Numbers in Context**

This lesson is about adding and subtracting integers, both positive and negative, and about translating back and forth between these arithmetic operations and the context in which they arise. The context is that of an initial temperature, a temperature change, and a final temperature. Students are given two of these and they need to find the third. They are also given a sum or difference of integers and asked to match this calculation with one of several possible stories. Finally, they are given one set of cards with names of cities, some labeled with temperatures, and another set with “change of temperature arrows”. They need to connect cities with these arrows in order to obtain triples of cards where the initial temperature plus the temperature change results in the final temperature.

Students need to be able to add and subtract integers, reason about these operations in context, and understand that the temperature change equals the final temperature minus the initial temperature (and equivalent formulations of this statement). They are asked to work in small groups, explain their reasoning, and critique each other’s work.

## Unit 7.5 Expressions

**Drawing to Scale: *Designing a Garden***

In this lesson, students need to design a garden of given dimensions to include a number of desired features. Students are provided a map, which is a rectangle drawn to scale, where the garden needs to be designed. On this map, students need to design the layout and draw the desired features, making sure the drawings are of appropriate size.

First, students need to compute the scale of the map. They can do this by measuring the map and comparing the dimensions to the real-life dimensions given in the problem. A number of methods can be used to represent this scale, and some samples are provided for critique. The main concept in this part is the proportional relationship, given by the scale, between distances on the map and their corresponding real-life distances.

Once they have a scale for the map, students need to use proportional reasoning in order to make decisions about the size of each garden feature to be drawn on the map, given the desired real-life size or constraints on size of the feature. Some of these desired dimensions are irrelevant, some require students to use the area formula for a circle, and some require thinking about the relationship between the volume, base area and height of a prism. In addition, estimation skills are required to find a reasonable length for a roll of grass to use.

In order to access this lesson, students need familiarity with metric units (length, area and volume), a basic knowledge of proportional relationships, familiarity with scale factors, the area formula for a circle, and the ability to calculate the volume of a prism from base area and height. They need to apply knowledge of these concepts in the appropriate contexts.

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## Unit 7.6 Equations

**Steps to Solving Equations**

This is a lesson about linear equations in one variable. Students need to write down equations that can be used to solve given word problems in a variety of contexts. They also need to match equations with the stories they model. They have to solve these equations, displaying the steps and reasons for each step in logical order, and they have to look for alternative methods of solving the equations.

Students will need to understand how to use the distributive property, combine like terms, and perform the same operations to both sides of an equation. They need to understand why each step they choose makes the equation simpler. Students are asked to work in small collaborative groups, explain their reasoning, and critique each other's work.

## Unit 7.7 Critical Areas 1 & 2

**Increasing and Decreasing Quantities by a Percent**

The goal of this lesson is for students to understand the concept of increasing or decreasing a quantity by a percent, and how the new quantity can be obtained from the original quantity by multiplying by a single number. Students are led to explore and understand the relationship between this number and the percent change. They also need to go back and forth fluently between percents, decimals, and fractions. Fractions are very useful for understanding the relationship between two consecutive percent changes (one an increase and the other a decrease, or vice versa) that leave the original quantity unchanged. They are led to understand that the two numbers used in the multiplications that reflect these percent changes are reciprocals of each other. They will also discover that a percent increase followed by a decrease of the resulting quantity by the same percent always results in a final quantity that is smaller than the original quantity.

## Unit 7.9 Geometric Measurement

**Using Dimensions: Designing a Sports Bag**

In this lesson, students create scale models of two-dimensional figures that can be sewn together to produce a cylindrical sports bag with prescribed dimensions. They are told that they need a rectangle and two circles, but are not given the dimensions of each piece. They need to allow for seam allowances or hems around each piece of fabric, and they need to realize that the length of the rectangle (before hem is added) must match the prescribed circumference of the circular end pieces (before hem is added).

In order to achieve this task, students need to know the formula for the circumference of a circle, they need to use proportional reasoning when making a scale drawing, they need accuracy in measuring lengths Students also need to make reasonable decisions when rounding numbers, especially making sure that they round up when rounding down would lead to a problem, like the pieces not fitting well together.

Students are asked to explain and critique each other's solutions as well as three given sample solutions with different flaws and strengths. Finally, another question asks for the optimal arrangement of the three necessary pieces on a strip of fabric 1 yard wide in order to be able to buy the shortest possible piece.

**Maximizing Area: Gold Rush**

In this lesson, students are given a scenario where they need to obtain the largest possible plot of land by using a rope of fixed length and four stakes to make a rectangular plot. They need to realize that this problem asks for the rectangle of maximum area among all rectangles with a fixed perimeter. Students can find that the rectangle that maximizes the area is a square by considering several rectangles and organizing their data in a table, or they can actually prove this using the fact that $(a + b) \times (a - b)$ is less than $a \times a$ if $b$ is a real number (although neither the teacher notes nor the sample work shown mention this method).

Students then need to analyze the case in which multiple people join their ropes together to make a larger rectangle. This results in more area per person than individual plots, and students can find this result by checking a few cases, or they can realize that when you multiply the perimeter of a square by a constant, the area is multiplied by the square of that constant. Therefore, if $n$ people join ropes together, the area is multiplied by the square of $n$, leaving each person with $n$ times as much area as they would get on their own. This is an issue of similar figures and the behavior of length and area under scaling.

Students need to work collaboratively to create posters with their solutions. They need to analyze and critique each other's work as well as a few samples of student work.

## Unit 7.10 Critical Areas 1, 2, & 3

**Estimations and Approximations: The Money Munchers**

In this lesson, students need to estimate the height of 24,400 one-dollar bills stored under a bed, and also whether the bills would fit in a 14'' by 19'' by 7'' suitcase. They need to make reasonable estimates for the three dimensions of a dollar bill and for the dimensions of a bed, and use these estimates productively as they reason about the ways of stacking the money under a bed or in the given suitcase. They need to explain their chain of reasoning, which involves drawing some stacking models and proportional reasoning.

The students are given several sample solutions to the problem, with different strengths and flaws, which they are asked to analyze. Students' answer will vary, since their answers will depend on estimated sizes of a mattress and a dollar bill, but they should be able to defend their estimations and final answers.

## Unit 7.13 Critical Areas 1, 2, 3, & 4

**Estimating: Counting Trees**

In this lesson, students are given a diagram of a tree farm that has old and new trees. They need to estimate the number of each type. The tree farm has the shape of a square, with 50 rows and 50 columns, each space has a new tree, an old tree, or a clearing. The number of old trees is significantly larger than the number of new trees. In order to solve this problem, students must be able to select a reasonable sampling method and use proportional reasoning to scale up their sample to make estimates about the entire tree farm. For the sampling, they need to realize that they should choose at least two areas of equal size and preferably not too close together. They should also take into account the gaps with no trees.

Students should understand that their computations yield estimates, and not the exact number of trees. They are also asked to explain their assumptions, choice of sampling method, and computations involving proportional reasoning. They are asked to interpret and critique each other's solutions and three given sample solutions, paying attention to any flaws in the assumptions made (for example, assuming an equal number of each type of tree), or in the sampling method.

**Evaluating Statements About Probability**

This is a lesson about the probability of events with a finite number of possible outcomes. Students are asked to evaluate many different statements about the probability of certain events, and they need to compute a few probabilities. For the latter, students need to be able to count and organize all possible outcomes, count the favorable outcomes, decide whether or not the outcomes all have equal probability, and form the correct fraction or use proportional reasoning to find the probability.

The lesson specifically targets several typical misconceptions about the probability of events. Some of these misconceptions, which students need to unmask and refute, are that previous events somehow can affect later events in contexts where the events are in fact independent; that all possible outcomes are equally likely in contexts where they are not; that some events that seem special are less likely than others that appear more representative in contexts where all outcomes are equally likely; or that probabilities give the proportion of outcomes that must actually occur in an experiment.

# Grade 8 Course Outline

The colored bars reflect the weighting of the standards within each unit based on the Cluster Emphases given by the PARCC and Smarter Balanced Consortia.

| Content Area | Formative Assessment Lessons | # of Days |
|---|---|---|
| Unit 8.0 Introduction: Mathematical Investigations | | 5 |
| Unit 8.1 Numerical Expressions<br>8.EE.1, 2, 3, 4 | Estimating Length Using Scientific Notation<br>Generalizing Patterns The Difference of Two Squares | 10 |
| Unit 8.2 Proportional Relationships, Lines, and Equations<br>8.EE.5, 6 | Lines and Linear Equations | 10 |
| Unit 8.3 Linear Equations and Systems of Linear Equations<br>8.EE.7, 8 | Solving Linear Equations in One Variable<br>Classifying Solutions to Systems of Equations | 10 |
| Unit 8.4 Bivariate Data<br>8.SP.1, 2, 3, 4 | | 10 |
| Unit 8.5 Critical Area 1<br>8.EE / 8.SP | Solving Real-Life Problems: Baseball Jerseys | 11 |
| Unit 8.6 Functions<br>8.F.1, 2, 3 / 8.F.4, 5 | Interpreting Distance-Time Graphs | 15 |
| Unit 8.7 Critical Areas 1 & 2<br>8.EE; 8.F.1, 2, 3 / 8.F.4, 5; 8.SP | Modeling Situations with Linear Equations | 15 |
| Unit 8.8 Rational and Irrational Numbers<br>8.NS.1, 2 | Repeating Decimals | 10 |
| Unit 8.9 Transformations<br>8.G.1, 2, 3, 4, 5 | Identifying Similar Triangles | 10 |
| Unit 8.10 Pythagorean Theorem<br>8.G.6, 7, 8 | The Pythagorean Theorem: Square Areas | 10 |
| Unit 8.11 Volume of Cones, Cylinders, and Spheres<br>8.G.9 | Modeling: Making Matchsticks | 10 |
| Unit 8.12 Critical Areas 1, 2, & 3<br>8.EE; 8.F.1, 2, 3; 8.G.1-8 / 8.F.4, 5; 8.SP / 8.G.9 | | 18 |
| Total Days | | 144 |

The Formative Assessment Lessons are available at [http://map.mathshell.org/materials/lessons.php](http://map.mathshell.org/materials/lessons.php)

# Formative Assessment Lessons Aligned to Grade 8

## Unit 8.1 Numerical Expressions

### Estimating Length Using Scientific Notation

This lesson asks students to translate back and forth between numbers written in decimal and scientific notation, and to order such numbers given either representation. It also asks students to estimate the length of several objects and to compare their relative size. Students are given a set of cards with length measurements (in meters). Each measurement (except for two blank cards that students must fill in themselves) appears on two cards, once as a decimal and once in scientific notation. They are also given another set of cards with drawings of various objects of a wide range of lengths. They need to match the two sets. There is an additional set of cards with multipliers and arrows for students to make multiplicative comparisons among the lengths of the different objects. Students show their matches and comparisons in posters, and they explain and critique all the posters.

In order to do the required tasks, students need to know how to manipulate numbers in decimal and scientific notation and they need to make reasonable estimates about lengths of objects.

### Generalizing Patterns: The Difference of Two Squares

The underlying mathematics to this lesson is number theory, or more precisely, Diophantine equations. There is just one question to consider: which integers can be written as the difference of two squares (of integers)? One can answer this question using tools that students do not know in 8$^{th}$ grade (like modular arithmetic – considering integers modulo 4 in particular). But the purpose of the lesson is not to obtain a complete solution with proof. Instead, the goal is for students to experiment, organize their work and findings, notice patterns and structure, make conjectures, test them, revise them if necessary, and explain their statements. They should notice patterns as they organize their results in tables, and they should attempt to prove that these patterns generalize by considering and manipulating quadratic expressions or by interpreting them via area models.

For students to access this lesson, they need organizational skills, ability to detect patterns and structure, fluency with integer arithmetic, manipulation of quadratic expressions involving variables, and some sophistication in their mathematical reasoning ability: they need to have some idea of what is a complete proof, what is needed to reject a conjecture, and how much evidence is reasonable to formulate one.

## Unit 8.2 Proportional Relationships, Lines, and Equations

### Lines and Linear Equations

This lesson asks for students to interpret and create mathematical models of linear relationships in two contexts. In the first context, two runners race each other. They run at constant speeds. The slower runner starts closer to the finish line but nonetheless is overtaken by the faster runner. The second context is about water flowing out of a container and into a container directly below.

(NOTE: This second context is problematic because the physics of the situation makes this a non-linear problem in real life. The water would flow faster at the beginning and slower as time goes on. However students are asked to *assume* that the height of the water will change linearly with time in both containers.)

For the racing context, students need to interpret a graph showing distance vs. time for the two runners. They need to understand the meaning of the slope of the lines (speed), the point of intersection (time and distance when the lead changes), intercepts (distance at the beginning), and x value when distance is 70 yards (time it takes to finish). They are asked to write down equations describing the runners' graphs. They also need to produce a new graph of the same situation, but in which the distance is measured from the finish line instead of the starting point of the faster runner.

For the water flowing context, students need to match graphs for each of the containers and descriptions of the containers with initial amounts of water and specified rates of flow. Some graphs or drawings are missing numerical information, the students need to be able to supply the missing numbers for these case.

The goal of this lesson is for students to understand the relationship between constant rate problems and linear graphs (the slope is the rate, the starting point is the intercept). Students should also take away from this lesson that certain real-life situations can be modeled as constant rate problems and described by linear equations.

Students are asked to work collaboratively, to analyze other groups' work, and to discuss any discrepancies in methods or results, arguing critically and clearly.

## Unit 8.3 Linear Equations & Systems of Linear Equations

### Solving Linear Equations in One Variable

In this lesson, students need to solve linear equations in one variable. They need to classify many different equations as "always true", "sometimes true" or "never true" according to whether the equations are satisfied by all numbers, exactly one number, or no numbers. In each case, they need to explain their answers. In particular, they need to find the unique solution for every "sometimes true" equation and find at least one value of *x* (the variable) that does not satisfy the equation. They are not asked to prove that the "sometimes true" equations have one unique solution. They simply need to find a solution (which happens to be unique, of course) and a value that does not satisfy the equation.

Students need to be able to add monomials (usually referred to as collecting like terms), add, subtract, multiply or divide the same quantity (including negative numbers) from both sides of an equation, and use the distributive property. They need to explain their work, analyze given sample student work, and collaborate in small groups.

Note: In these Course Outlines, we replace the commonly used expression "collect like terms" with "adding monomials" because the latter is more easily understood by English Language Learners.

### Classifying Solutions to Systems of Equations

This is a lesson about linear equations and systems of two linear equations in two variables. Given a table of values, an equation, or a graph of a linear relationship in two variables, students need to produce the other two representations. They need to understand that the graph of an equation consists of all the points in the Cartesian plane satisfying the equation. They need to decide which tables of values agree with a given equation or graph, be able to graph a line given an equation or table, and find a linear equation given a graph or table of values. They also need to determine when a system of two linear equations has no simultaneous solutions, one unique solution, or infinitely many solutions. They should understand why these are the only possibilities. They should be able to tell how many solutions there are both from the equations of the lines and from their graphs. They need to interpret each case in terms of the slopes of the two lines and whether or not two equations are equivalent (meaning their graph is the same line). They must know that two different parallel lines have equal slopes and never meet, and that two non-parallel lines meet at a unique point. When there is a unique solution, they need to be able to find it, graphically and algebraically.

Students grouped in pairs are given cards containing partial information about a linear relationship. Each card must be completed so it will have a table of values, an equation, and a graph. After the cards are completed and put in a poster, students are given several new cards saying “no common solutions,” “infinitely many common solutions,” or “one common solution when $x$ = ___ , $y$ = ___.” Students need to connect several different pairs of cards of linear relationships in their posters with these new cards according to the number of solutions each relationship has. When there is a unique solution, students must complete the new card by finding it.

## Unit 8.5 Critical Area 1

**Solving Real-Life Problems: Baseball Jerseys**

This is a lesson about comparing options for buying jerseys at two companies. One company charges a fixed price per jersey. The other company charges a smaller fixed price per jersey plus an initial set-up fee. Therefore, for both companies, the price is a linear function of the number of jerseys ordered. Students need to decide which company to use for a specified number of jerseys (i.e., which company will cost less). They also need to find the range over which each company offers a better deal than the other, and the break-even point.

Solutions can involve making a table or chart with several data points, solving a linear inequality, solving a linear equation to find the break-even point, and then doing some reasoning or computations to find what company is cheaper over each side of this point, or graphing the total cost as a function of the number of jerseys. Interpreting this graph readily yields the break-even point and the range over which each company is cheaper. Students are also asked to develop a pricing model that, for any positive number of jerseys ordered, would charge a price that is not higher or lower than the two given pricing models.

In order to solve the tasks assigned, students need to be able to model the problem mathematically, interpret their models (tabular or graphical linear data), and solve linear equations or inequalities. Students are also asked to explain and critique each other’s solutions, as well as four sample solutions that are provided for discussion.

## Unit 8.6 Functions

### Interpreting Distance–Time Graphs

In this lesson students are asked to interpret distance–time graphs, and match them with possible scenarios about a person's displacement from a fixed point (home) along a straight path (road). They also need to match these to different tables of values for time and distance from home.

In order to complete this lesson, students need to understand that the slope of the graph at any point is the speed at that moment, so that larger slopes correspond to faster movement. They also need to understand that a negative slope indicates movement toward home. The graphs are for the most part piecewise linear (there are two exceptions). For these graphs, students need to understand that each linear segment corresponds to a time interval over which the speed is constant. One of the graphs has a vertical line segment. Students need to realize that this graph cannot reflect any real situation, since a person cannot be at more than one place at a given moment.

## Unit 8.7 Critical Areas 1 & 2

### Modeling Situations with Linear Equations

This lesson involves modeling and interpreting situations involving four variable quantities, and fixing two at a time to see how the other variables depend on each other. In each of the contexts supplied in this lesson, these relationships will be linear. Mathematically, the situations are identical, but they have different contexts. For example, one of the contexts deals with the profit, $p$, made by selling a number of candles, $n$, at a unit price, $u$, when there is a fixed cost, $k$. Students need to graph $p$ as a function of $n$ if $u$ and $k$ are fixed. They also need to solve for each of the four variables in terms of the other three to produce equations governing the entire relationship.

Students need to manipulate symbolic expressions to solve equations, make graphs, and find specific values. They need to interpret the meaning of expressions and values, such as the meaning of the $x$-intercept of *the* $p$ vs. $n$ function above, or the meaning of the expression $u \cdot n$.

## Unit 8.8 Rational and Irrational Numbers

### Repeating Decimals

In this lesson, students need to translate back and forth between fractions and their decimal representations, with an emphasis on repeating decimals. Students also need to understand what happens when these decimals are multiplied by powers of 10, and select the appropriate powers to set up simple linear equations in order to obtain the fraction with the same value as a given decimal.

Note: this method of setting up a simple equation that eliminates the infinitely repeating decimal cannot be formally justified at this stage, since a true justification requires working with infinite series. Still, the subtraction needs to be justified. This should probably be made clear, even if only in passing, while assuring students that the method does indeed work and can be justified.

Some of the work needs to be done without a calculator, while in other parts students are allowed to use calculators. In small groups, students also need to match cards from three lists (and make a poster with the matches they find): one has fractions, one has decimals, and one has linear equations that would be useful to translate from the decimal to the fraction. Sometimes the third matching card of a triple is missing, and students need to create the missing card. Students are asked to explain their work to others and to critique each other's arguments and results.

## Unit 8.9 Transformations

**Identifying Similar Triangles**

This is a geometry lesson about angles and similar triangles. The given diagrams are not necessarily drawn to scale, students need to determine which angle measures can, and which cannot, be computed from the given information. They also have to decide whether or not two given triangles are similar.

Students need to realize that a diagram not drawn to scale can be misleading (lines that seem parallel may not be, or vice-versa, and the relative size of angles may not correspond to the visual size of the diagram), so they can only assume what is explicitly given. For this, they need to know the notation for parallel lines and for congruent angles and segments.

The knowledge that students need for this lesson includes the angle sum and the exterior angle theorems for triangles, the angle-angle criterion for similarity of triangles, the fact that the measure of an angle formed by adjacent angles is the sum of these angles' measures, that supplementary angles have a measure sum of 180 degrees, and facts about angles formed by a transversal to two lines, including how to determine whether or not the lines are parallel.

## Unit 8.10 Pythagorean Theorem

**The Pythagorean Theorem: Square Areas**

In this lesson, students explore and prove the Pythagorean theorem by considering the area of a square inscribed and rotated inside another square. Both squares have vertices on a lattice (grid paper), with consecutive horizontal and vertical dots separated evenly by intervals of 1 unit.

Students can compute the area of the inscribed square by several different methods: using the area of right triangles, subtracting from the large square, dissecting the inscribed square, or rearranging congruent pieces.  Pairs of students are subsequently asked to analyze all possible areas of lattice squares with a given tilt. For this, they need to collect and organize their data, find patterns, and share the results with the rest of the class. As a whole class, the results from the different pairs are collected to conclude that the areas are all sums of two squares of whole numbers.

Students are asked to consider what areas can or cannot result from tilted lattice squares of any tilt (a question whose solution requires more sophistication than students can be expected to handle, but this is explicitly noted in the teacher materials with a brief discussion of the matter). Finally, they are led to a proof of the Pythagorean theorem, and hence to the length of the hypotenuse of a right triangle, by a method of rearranging the four congruent right triangles between the large and the inscribed square.

Students are given sample student work, which is not very detailed, and asked to make sense of the reasoning behind these sample methods by providing the missing details. They need to collaborate and critique each other's work.

## Unit 8.11 Volume of Cones, Cylinders, and Spheres

**Modeling: Making Matchsticks**

In this lesson, students need to estimate how many matchsticks can be made from a pine tree. The dimensions of a matchstick are given in inches, and a sketch of the tree is provided with measurements (height and diameter at the base) in feet. Students are also provided with a sheet containing area and volume formulas for several 2-D and 3-D geometric objects. Students need to model the tree as a geometric object, use appropriate formulas, make and justify their estimates and calculations, and round their answers to a reasonable degree of accuracy.

They need to pay attention to units and how volume in particular is computed after a change of units, because in order to obtain an answer, they need to compare the volume of a matchstick to the volume of the tree in the same units.

Students need to analyze sample solutions and evaluate their own and each other's work. They need to work collaboratively and communicate with each other clearly. They need to be able to explain their solutions thoroughly, explicitly noting their estimates and justifying their formulas and computations.